Elliptic Boundary Value Problems In The Spaces Of Distributions 1st Edition

This monograph systematically treats a theory of elliptic boundary value problems in domains without singularities and in domains with conical or cuspidal points. This exposition is self-contained and a priori requires only basic knowledge of functional analysis. Restricting to boundary value problems formed by differential operators and avoiding the use of pseudo-differential operators makes the book accessible for a wider readership. The authors concentrate on fundamental results of the theory: estimates for solutions in different function spaces, the Fredholm property of the operator of the boundary value problem, regularity assertions and asymptotic formulas for the solutions near singular points. A special feature of the book is that the solutions of the boundary value problems are considered in Sobolev spaces of both positive and negative orders. Results of the general theory are illustrated by concrete examples. The book may be used for courses in partial differential equations.

For the first time in the mathematical literature, this two-volume work introduces a unified and general approach to the subject. To a large extent, the book is based on the authors' work, and has no significant overlap with other books on the theory of elliptic boundary value problems

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Boundary Value Problems for Elliptic Systems

Nonlinear Elliptic Boundary Value Problems and Their Applications

Elliptic Problems in Nonsmooth Domains

Elliptic Boundary Value Problems in Domains with Point Singularities

Elliptic Problems in Domains with Piecewise Smooth Boundaries

Hierarchical matrices are an efficient framework for large-scale fully populated matrices arising, e.g., from the finite

element discretization of solution operators of elliptic boundary value problems. In addition to storing such matrices, approximations of the usual matrix operations can be computed with logarithmic-linear complexity, which can be exploited to setup approximate preconditioners in an efficient and convenient way. Besides the algorithmic aspects of hierarchical matrices, the main aim of this book is to present their theoretical background. The book contains the existing approximation theory for elliptic problems including partial differential operators with nonsmooth coefficients. Furthermore, it presents in full detail the adaptive cross approximation method for the efficient treatment of integral operators with non-local kernel functions. The theory is supported by many numerical experiments from real applications.
The aim of the series is to present new and important developments in pure and applied mathematics. Well established in the community over two decades, it offers a large library of mathematics including several important classics. The volumes supply thorough and detailed expositions of the methods and ideas essential to the topics in question. In addition, they convey their relationships to other parts of mathematics. The series is addressed to advanced readers wishing to thoroughly study the topic.
For the first time in the mathematical literature this two-volume work introduces a unified and general approach to the asymptotic analysis of elliptic boundary value problems in singularly perturbed domains. While the first volume is devoted to perturbations of the boundary near isolated singular points, the second volume treats singularities of the boundary in higher dimensions as well as nonlocal perturbations. At the core of this work are solutions of elliptic boundary value problems by asymptotic expansion in powers of a small parameter that characterizes the perturbation of the domain. In particular, it treats the important special cases of thin domains, domains with small cavities, inclusions or ligaments, rounded corners and edges, and problems with rapid oscillations of the boundary or the coefficients of the differential operator. The methods presented here capitalize on the theory of elliptic boundary value problems with nonsmooth boundary that has been developed in the past thirty years. Moreover, a study on the homogenization of differential and difference equations on periodic grids and lattices is given. Much attention is paid to concrete problems in mathematical physics, particularly elasticity theory and electrostatics. To a large extent the work is based on the authors' work and has no significant overlap with other books on the theory of elliptic boundary value problems.
Methods for Analysis of Nonlinear Elliptic Boundary Value Problems
Partial Differential Equations and Boundary Value Problems
Elliptic Boundary Value Problems with Fractional Regularity Data: The First Order Approach

Elliptic Boundary Value Problems and Construction of LP-Strong Feller Processes with Singular Drift and Reflection
Elliptic Boundary Value Problems on Corner Domains

This book presents a unified theory of the Finite Element Method and the Boundary Element Method for a numerical solution of second order elliptic boundary value problems. This includes the solvability, stability, and error analysis as well as efficient methods to solve the resulting linear systems. Applications are the potential equation, the system of linear elastostatics and the Stokes system. While there are textbooks on the finite element method, this is one of the first books on Theory of Boundary Element Methods. It is suitable for self study and exercises are included.

This book focuses on the analysis of eigenvalues and eigenfunctions that describe singularities of solutions to elliptic boundary value problems in domains with corners and edges. The authors treat both classical problems of mathematical physics and general elliptic boundary value problems. The volume is divided into two parts: the first is devoted to the power-logarithmic singularities of solutions to classical boundary value problems of mathematical physics. The second deals with similar singularities for higher order elliptic equations and systems. Chapter 1 collects basic facts concerning operator pencils acting in a pair of Hilbert spaces. Related properties of ordinary differential equations with constant operator coefficients are discussed and connections with the theory of general elliptic boundary value problems in domains with conic vertices are outlined. New results are presented.Chapter 2 treats the Laplace operator as a starting point and a model for the subsequent study of angular and conic singularities of solutions. Chapter 3 considers the Dirichlet boundary condition beginning with the plane case and turning to the space problems. Chapter 4 investigates some mixed boundary conditions. The Stokes system is discussed in Chapters 5 and 6, and Chapter 7 concludes with the Dirichlet problem for the polyharmonic operator. Chapter 8 studies the Dirichlet problem for general elliptic differential equations of order $2m$ in an angle.In Chapter 9, an asymptotic formula for the distribution of eigenvalues of operator pencils corresponding to general elliptic boundary value problems in an angle is obtained. Chapters 10 and 11 discuss the Dirichlet problem for elliptic systems of differential equations of order 2 in an n-dimensional cone. Chapter 12 studies the Neumann problem for general elliptic systems, in particular with eigenvalues of the corresponding operator pencil in the strip $\mid {\Re} \lambda - m + /2n \mid \leq 1/2$. It is shown that only integer numbers contained in this strip are eigenvalues. Applications are placed within chapter introductions and as special sections at the end of chapters. Prerequisites include standard PDE and functional analysis courses.

The theory of nonlinear elliptic equations is currently one of the most actively developing branches of the theory of partial differential equations. This book investigates boundary value problems for nonlinear elliptic equations of arbitrary order. In addition to monotone operator methods, a broad range of applications of topological methods to nonlinear differential equations is presented: solvability, estimation of the number of solutions, and the branching of solutions of nonlinear equations. Skrypnik establishes, by various procedures, a priori estimates and the regularity of solutions of nonlinear elliptic equations of arbitrary order. Also covered are methods of homogenization of nonlinear elliptic problems in perforated domains. The book is suitable for use in graduate courses in differential equations and nonlinear functional analysis.

Lectures on Elliptic Boundary Value Problems

Boundary Value Problems for Second Order Elliptic Equations

Harmonic Analysis Techniques for Second Order Elliptic Boundary Value Problems

Elliptic Boundary Value Problems of Second Order in Piecewise Smooth Domains

Elliptic Boundary Value Problems

The theory of boundary value problems for elliptic systems of partial differential equations has many applications in mathematics and the physical sciences. The aim of this book is to "algebraize" the index theory by means of pseudo-differential operators and new methods in the spectral theory of matrix polynomials. This latter theory provides important tools that will enable the student to work efficiently with the principal symbols of the elliptic and boundary operators on the boundary. Because many new methods and results are introduced and used throughout the book, all the theorems are proved in detail, and the methods are well illustrated through numerous examples and exercises. This book is ideal for use in graduate level courses on partial differential equations, elliptic systems, pseudo-differential operators, and matrix analysis.

The book contains a systematic treatment of the qualitative theory of elliptic boundary value problems for linear and quasilinear second order equations in non-smooth domains. The authors concentrate on the following fundamental results: sharp estimates for strong and weak solutions, solvability of the boundary value problems, regularity assertions for solutions near singular points. Key features: * New the Hardy – Friedrichs – Wirtinger type inequalities as well as new integral inequalities related to the Cauchy problem for a differential equation. *

Precise exponents of the solution decreasing rate near boundary singular points and best possible conditions for this. * The question about the influence of the coefficients smoothness on the regularity of solutions. * New existence theorems for the Dirichlet problem for linear and quasilinear equations in domains with conical points. * The precise power modulus of continuity at singular boundary point for solutions of the Dirichlet, mixed and the Robin problems. * The behaviour of weak solutions near conical point for the Dirichlet problem for m – Laplacian. * The behaviour of weak solutions near a boundary edge for the Dirichlet and mixed problem for elliptic quasilinear equations with triple degeneration. * Precise exponents of the solution decreasing rate near boundary singular points and best possible conditions for this. * The question about the influence of the coefficients smoothness on the regularity of solutions. * New existence theorems for the Dirichlet problem for linear and quasilinear equations in domains with conical points. * The precise power modulus of continuity at singular boundary point for solutions of the Dirichlet, mixed and the Robin problems. * The behaviour of weak solutions near conical point for the Dirichlet problem for m - Laplacian. * The behaviour of weak solutions near a boundary edge for the Dirichlet and mixed problem for elliptic quasilinear equations with triple degeneration.

This volume endeavours to summarise all available data on the theorems on isomorphisms and their ever increasing number of possible applications. It deals with the theory of solvability in generalised functions of general boundary-value problems for elliptic equations. In the early sixties, Lions and Magenes, and Berezansky, Krein and Roitberg established the theorems on complete collection of isomorphisms. Further progress of the theory was connected with proving the theorem on complete collection of isomorphisms for new classes of problems, and hence with the development of new methods to prove these theorems. The theorems on isomorphisms were first established for elliptic equations with normal boundary conditions. However, after the Noetherian property of elliptic problems was proved without assuming the normality of the boundary expressions, this became the natural way to consider the problems of establishing the theorems on isomorphisms for general elliptic problems. The present author's method of solving this problem enabled proof of the theorem on complete collection of isomorphisms for the operators generated by elliptic boundary-value problems for general

systems of equations. Audience: This monograph will be of interest to mathematicians whose work involves partial differential equations, functional analysis, operator theory and the mathematics of mechanics.

Elliptic Boundary Value Problems with Indefinite Weights, Variational Formulations of the Principal Eigenvalue, and Applications

Spectral Problems Associated with Corner Singularities of Solutions to Elliptic Equations

Numerical Approximation Methods for Elliptic Boundary Value Problems

Singularities in Elliptic Boundary Value Problems and Elasticity and Their Connection with Failure Initiation

Smoothness and Asymptotics of Solutions

This introductory and self-contained book gathers as much explicit mathematical results on the linear-elastic and heat-conduction solutions in the neighborhood of singular points in two-dimensional domains, and singular edges and vertices in three-dimensional domains. These are presented in an engineering terminology for practical usage. The author treats the mathematical formulations from an engineering viewpoint and presents high-order finite-element methods for the computation of singular solutions in isotropic and anisotropic materials, and multi-material interfaces. The proper interpretation of the results in engineering practice is advocated, so that the computed data can be correlated to experimental observations. The book is divided into fourteen chapters, each containing several sections. Most of it (the first nine Chapters) addresses two-dimensional domains, where only singular points exist. The solution in a vicinity of these points admits an asymptotic expansion composed of eigenpairs and associated generalized flux/stress intensity factors (GFIFs/GSIFs), which are being computed analytically when possible or by finite element methods otherwise. Singular points associated with weakly coupled thermoelasticity in the vicinity of singularities are also addressed and thermal GSIFs are computed. The computed data is important in engineering practice for predicting failure initiation in brittle material on a daily basis. Several failure laws for two-dimensional domains with V-notches are presented and their validity is examined by comparison to experimental observations. A sufficient simple and reliable condition for predicting failure initiation (crack formation) in micron level electronic devices, involving singular points, is still a topic of active research and interest, and is addressed herein. Explicit singular solutions in the vicinity of vertices and edges in three-dimensional domains are provided in the remaining five chapters. New methods for the computation of generalized edge flux/stress intensity functions along singular edges are presented and demonstrated by several example problems from the field of fracture mechanics; including anisotropic domains and bimaterial interfaces. Circular edges are also presented and the author concludes with some remarks on open questions. This well illustrated book will appeal to both applied mathematicians and engineers working in the field of fracture mechanics and singularities.

A marriage of the finite-differences method with variational methods for solving boundary-value problems, the finite-element method is superior in many ways to finite-differences alone. This self-contained text for advanced undergraduates and graduate students is intended to imbed this combination of methods into the framework of functional analysis and to explain its applications to approximation of nonhomogeneous boundary-value problems for elliptic operators. The treatment begins with a summary of the main results established in the book. Chapter 1 introduces the variational method and the finite-difference method in the simple case of second-order differential equations. Chapters 2 and 3 concern abstract approximations of Hilbert spaces and linear operators, and Chapters 4 and 5 study finite-element approximations of Sobolev spaces. The remaining four chapters consider several methods for approximating nonhomogeneous boundary-value problems for elliptic operators.

Focussing on the interrelations of the subjects of Markov processes, analytic semigroups and elliptic boundary value problems, this monograph provides a careful and accessible exposition of functional methods in stochastic analysis. The author studies a class of boundary value problems for second-order elliptic differential operators which includes as particular cases the Dirichlet and Neumann problems, and proves that this class of boundary value problems provides a new example of analytic semigroups both in the Lp topology and in the topology of uniform convergence. As an application, one can construct analytic semigroups corresponding to the diffusion phenomenon of a Markovian particle moving continuously in the state space until it "dies", at which time it reaches the set where the absorption phenomenon occurs. A class of initial-boundary value problems for semilinear parabolic differential equations is also considered. This monograph will appeal to both advanced students and researchers as an introduction to the three interrelated subjects in analysis, providing powerful methods for continuing research.

Introductory Numerical Analysis of Elliptic Boundary Value Problems

Partial Differential Equations IX

A Means to Efficiently Solve Elliptic Boundary Value Problems

SET

Lecture Notes on Pseudo-differential Operators and Elliptic Boundary Value Problems

This book, which is based on several courses of lectures given by the author at the Independent University of Moscow, is devoted to Sobolev-type spaces and boundary value problems for linear *elliptic partial differential equations. Its main focus is on problems in non-smooth (Lipschitz) domains for strongly elliptic systems. The author, who is a prominent expert in the theory of linear partial differential equations, spectral theory and pseudodifferential operators, has included his own very recent findings in the present book. The book is well suited as a modern graduate textbook, utilizing a thorough and clear format that strikes a good balance between the choice of material and the style of exposition. It can be used both as an introduction to recent advances in elliptic equations and boundary value problems and as a valuable survey and reference work. It also includes a good deal of new and extremely useful material not available in standard textbooks to date. Graduate and post-graduate students, as well as specialists working in the fields of partial differential equations, functional analysis, operator theory and mathematical physics will find this book particularly valuable.*

A numerical method for solving elliptic boundary value problems in unbounded domains is described. An artificial boundary near infinity, say a sphere GAMMA/sub R/ of large radius R, and an approximate local boundary condition on GAMMA/sub R/ are introduced. A finite element method is then employed to discretize this approximate problem. Since R must be large in order to estimate the error due to the artificial boundary, the resulting system of linear equations is usually quite large. A description is given on how to reduce the number of linear equations to the asymptotically optimal amount, while optimal error estimates and the sparseness of the matrix are preserved, by grading the mesh systematically in such a way that the element mesh sizes are increased as the distance from the origin increases. Some numerical results are given. 1 table. (RWR).

In recent years, there has been a great deal of activity in the study of boundary value problems with minimal smoothness assumptions on the coefficients or on the boundary of the domain in question. These problems are of interest both because of their theoretical importance and the implications for applications, and they have turned out to have profound and fascinating connections with many areas of analysis. Techniques from harmonic analysis have proved to be extremely useful in these studies, both as concrete tools in establishing theorems and as models which suggest what kind of result might be true. Kenig describes these developments and connections for the study of classical boundary value problems on Lipschitz domains and for the corresponding problems for second order elliptic equations in divergence form. He also points out many interesting problems in this area which remain open.

Asymptotic Theory of Elliptic Boundary Value Problems in Singularly Perturbed Domains
Boundary Value Problems and Markov Processes
Wavelet Methods — Elliptic Boundary Value Problems and Control Problems
Nonlinear Elliptic Boundary Value Problems

Elliptic Boundary Value Problems With Indefinite Weights presents a unified approach to the methodologies dealing with eigenvalue problems involving indefinite weights. The principal eigenvalue for such problems is characterized for various boundary conditions. Such characterizations are used, in particular, to formulate criteria for the persistence and extinctions of populations, and applications of the formulations obtained can be quite extensive.

Elliptic Boundary Value Problems on Corner DomainsSmoothness and Asymptotics of SolutionsSpringer

This accessible monograph covers higher order linear and nonlinear elliptic boundary value problems in bounded domains, mainly with the biharmonic or poly-harmonic operator as leading principal part. It provides rapid access to recent results and references.

Elliptic Boundary Value Problems in the Spaces of Distributions
Volume I
Finite and Boundary Elements
Polyharmonic Boundary Value Problems
Asymptotic Theory of Elliptic Boundary Value Problems in Singularly Perturbed Domains Volume II

A co-publication of the AMS and Centre de Recherches Math é matiques In this monograph the authors study the well-posedness of boundary value problems of Dirichlet and Neumann type for elliptic systems on the upper half-space with coefficients independent of the transversal variable and with boundary data in fractional Hardy – Sobolev and Besov spaces. The authors use the so-called " first order approach " which uses minimal assumptions on the coefficients and thus allows for complex coefficients and for systems of equations. This self-contained exposition of the first order approach offers new results with detailed proofs in a clear and accessible way and will become a valuable reference for graduate students and researchers working in partial differential equations and harmonic analysis.

This EMS volume gives an overview of the modern theory of elliptic boundary value problems, with contributions focusing on differential elliptic boundary problems and their spectral properties, elliptic pseudodifferential operators, and general differential elliptic boundary value problems in domains with singularities.

This research monograph focusses on a large class of variational elliptic problems with mixed boundary conditions on domains with various corner singularities, edges, polyhedral vertices, cracks, slits. In a natural functional framework (ordinary Sobolev Hilbert spaces) Fredholm and semi-Fredholm properties of induced operators are completely characterized. By specially choosing the classes of operators and domains and the functional spaces used, precise and general results may be obtained on the smoothness and asymptotics of solutions. A new type of characteristic condition is introduced which involves the spectrum of associated operator pencils and some ideals of polynomials satisfying some boundary conditions on cones. The methods involve many perturbation arguments and a new use of Mellin transform. Basic knowledge about BVP on smooth domains in Sobolev spaces is the main prerequisite to the understanding of this book. Readers interested in the general theory of corner domains will find here a new basic theory (new approaches and results) as well as a synthesis of many already known results; those who need regularity conditions and descriptions of singularities for numerical analysis will find precise statements and also a means to obtain further one in many explicit situtations.

Approximation of Elliptic Boundary-Value Problems

Positivity Preserving and Nonlinear Higher Order Elliptic Equations in Bounded Domains

Sobolev Spaces, Their Generalizations and Elliptic Problems in Smooth and Lipschitz Domains

Volume II

Computational Methods for Mildly Nonlinear Elliptic Boundary Value Problems with Multiple Solutions

Diese Monographie spannt einen Bogen rund um die aktuelle Thematik Wavelets, um neueste Entwicklungen anhand aufeinander aufbauender Probleme darzustellen und das konzeptuelle Potenzial von Waveletmethoden für Partielle Differentialgleichungen zu demonstrieren.

This book, which is a new edition of a book originally published in 1965, presents an introduction to the theory of higher-order elliptic boundary value problems. The book contains a detailed study of basic problems of the theory, such as the problem of existence and regularity of solutions of higher-order elliptic boundary value problems. It also contains a study of spectral properties of operators associated with elliptic boundary value problems. Weyl's law on the asymptotic distribution of eigenvalues is studied in great generality. The material of the present book has been used for graduate-level courses at the University of Ia~i during the past ten years. It is a revised version of a book which appeared in Romanian in 1993 with the Publishing House of the Romanian Academy. The book focuses on classical boundary value problems for the principal equations of mathematical physics: second order elliptic equations (the Poisson equations), heat equations and wave equations. The existence theory of second order elliptic boundary value problems was a great challenge for nineteenth century mathematics and its development was marked by two decisive steps. Undoubtedly, the first one was the Fredholm proof in 1900 of the existence of solutions to Dirichlet and Neumann problems, which represented a triumph of the classical theory of partial differential equations. The second step is due to S. 1. Sobolev (1937) who introduced the concept of weak solution in partial differential equations and inaugurated the modern theory of boundary value problems. The classical theory which is a product ofthe nineteenth century, is concerned with smooth (continuously differentiable) sollutions and its methods rely on classical analysis and in particular on potential theory. The modern theory concerns distributional (weak) solutions and relies on analysis of Sob ole v spaces and functional methods. The same distinction is valid for the boundary value problems associated with heat and wave equations. Both aspects of the theory are present in this book though it is not exhaustive in any sense.

Hierarchical Matrices

Quaternionic Analysis and Elliptic Boundary Value Problems

Numerical Method for Solving Elliptic Boundary Value Problems in Unbounded Domains

Originally published: Boston: Pitman Advanced Pub. Program, 1985.