Adaptive Finite Element Methods For Differential Equations By Wolfgang Bangerth

In this thesis, we develop algorihms for the solution of inverse problems by the adaptive finite element method. In particular, we consider problems where distributed coefficients in partial differential equations are to be identified from measurements of the state variable. The ingredients for efficient algorithms are error estimates for various quantities, such as the misfit or the error in the coefficient, and adaptive finite element discretization strategies. We also discuss the numerical solution of the resulting discrete equations, as well as the incorporation of inequality constraints on the sought coefficient. The efficiency of the proposed methods is verified at a number of synthetic examples.
A new approach to error control and mesh adaptivity is described for the discretization of optimal control problems governed by (elliptic) partial differential equations. The Lagrangian formalism yields the first-order necessary optimality condition in form of an indefinite boundary value problem which is approximated by an adaptive Galerkin finite element method. The mesh design in the resulting reduced models is controlled by residual-based a posteriori error estimates. These are derived by duality arguments employing the cost functional of the optimization problem for controlling the discretization error. In this case, the computed state and co-state variables can be used as sensitivity factors multiplying the local cell-residuals in the error estimators. This results in a generic and efficient algorithm for mesh adaptation within the optimization process. Applications of the developed method are boundary control problem models governed by Ginzburg-Landau equations (superconductivity in semi-conductors), by Navier-Stokes equations, and by the Boussinesq viscosity model (flow with temperature transport for zero gravitation). Computations with more than 2 million unknowns were performed.

ADAPTIVE FINITE ELEMENT ANALYS

Adaptive Finite Element Methods for Computing Nonstationary Incompressible Flows

With Applications in Aerodynamics

Long-time integration. V

High Performance Adaptive Finite Element Methods

With a focus on 1D and 2D problems, the first volume of Computing with hp-ADAPTIVE FINITE ELEMENTS prepared readers for the concepts and logic governing 3D code and implementation. Taking the next step in hp technology, Volume II Frontiers: Three-Dimensional Elliptic and Maxwell Problems with Applications presents the theoretical foundations of the 3D hp algorithm and provides numerical results using the 3Dhp code developed by the authors and their colleagues. The first part of the book focuses on fundamentals of the 3D theory of hp methods as well as issues that arise when the code is implemented. After a review of boundary-value problems, the book examines exact hp sequences, projection-based interpolation, and De Rham diagrams. It also presents the 3D version of the automatic hp-adaptivity package, a two-grid solver for highly anisotropic hp meshes and goal-oriented Krylov iterations, and a parallel implementation of the 3D code. The second part explores several recent projects in which the 3Dhp code was used and illustrates how these applications have greatly driven the development of 3D hp technology. It encompasses acoustic and electromagnetic (EM) scattering problems, an analysis of complex structures with thin-walled components, and challenging simulations of logging tools. The book concludes with a look at the future of hp methods. Spearheaded by a key developer of this technology with more than 20 years of research in the field, this self-contained, comprehensive resource will help readers overcome the difficulties in coding hp-adaptive elements.

This book is a collection of lecture notes for the CIME course on "Multiscale and Adaptivity: Modeling, Numerics and Applications," held in Cetraro (Italy), in July 2009. Complex systems arise in several physical, chemical, and biological processes, in which length and time scales may span several orders of magnitude. Traditionally, scientists have focused on methods that are particularly applicable in only one regime, and knowledge of the system on one scale has been transferred to another scale only indirectly. Even with modern computer power, the complexity of such systems precludes their being treated directly with traditional tools, and new mathematical and computational instruments have had to be developed to tackle such problems. The outstanding and internationally renowned lecturers, coming from different areas of Applied Mathematics, have themselves contributed in an essential way to the development of the theory and techniques that constituted the subjects of the courses.

Unified Multilevel Adaptive Finite Element Methods for Elliptic Problems

Adaptive Finite Element Methods for Optimization Problems

Adaptive Finite Element Methods for Direct and Inverse Problems in Nonlinear Solid Mechanics

Adaptive Finite Element Methods for Elliptic Equations

Adaptive Finite Element Methods for Small Strain Elasto-plasticity

Adaptive Finite Element Methods for Differential EquationsBirkhäuser

In this thesis we discuss convergence theory for goal-oriented adaptive finite element methods for second order elliptic problems. We develop results for both linear nonsymmetric and semilinear problems. We start with a brief description of the finite element method applied to these problems and some basic error estimates. We then provide a detailed error analysis of the method as described for each problem. In each case, we establish convergence in the sense of the quantity of interest with a goal-oriented variation of the standard adaptive finite element method using residual-based indicators. In the linear case we establish the adjoint as the appropriate differential operator for the dual problem. We establish contraction of the quasi-error for each of the primal and dual problems yielding convergence in the quantity of interest. We follow these results with a complexity analysis of the method. In the semilinear case we introduce three types of linearized dual problems used to establish our results. We give a brief summary of a priori estimates for this class of problems. After establishing contraction results for the primal problem, we then provide additional estimates to show contraction of the combined primal and dual system, yielding convergence of the goal function. We support these results with some numerical experiments. Finally, we include an appendix outlining some common methods used in a posteriori error estimation and briefly describe iterative methods for solving nonlinear problems.

Volume II Frontiers: Three Dimensional Elliptic and Maxwell Problems with Applications

Error-controlled Adaptive Finite Elements in Solid Mechanics

Adaptive Finite and Boundary Element Methods

Adaptive Finite Element Methods for Parabolic Problems

Simulation and Optimal Control

The authors discuss a finite element method for solving initial-boundary value problems for vector systems of partial differential equations in one space dimension and time. The method automatically adjusts the computational mesh as the solution evolves in time so as to approximately minimize the local discretization error. They are thus able to calculate accurate solutions with fewer elements than would be necessary with a uniform mesh. This overall method contains two distinct steps: a solution step and a mesh selection step. They solve the partial differential equations using a finite element-Galerkin method on trapezoidal space-time-elements with either piecewise linear or cubic Hermits polynomial approximations. A variety of mesh selection strategies are discussed and analyzed. Results are presented for several computational examples.

These Lecture Notes have been compiled from the material presented by the second author in a lecture series ('Nachdiplomvorlesung') at the Department of Mathematics of the ETH Zurich during the summer term 2002. Concepts of 'self adaptivity' in the numerical solution of differential equations are discussed with emphasis on Galerkin finite element methods. The key issues are a posteriori er ror estimation and automatic mesh adaptation. Besides the traditional approach of energy-norm error control, a new duality-based technique, the Dual Weighted Residual method (or shortly D

WR method) for goal-oriented error estimation is discussed in detail. This method aims at economical computation of arbitrary quantities of physical interest by properly adapting the computational mesh. This is typically required in the design cycles of technical applications. For example, the drag coefficient of a body immersed in a viscous flow is computed, then it is minimized by varying certain control parameters, and finally the stability of the resulting flow is investigated by solving an eigenvalue problem. 'Goal-oriented' adaptivity is designed to achieve these tasks with minimal cost. The basics of the DWR method and various of its applications are described in the following survey articles: R. Rannacher [114], Error control in finite element computations. In: Proc. of Summer School Error Control and Adaptivity in Scientific Computing (H. Bulgak and C. Zenger, eds), pp. 247-278. Kluwer Academic Publishers, 1998. M. Braack and R. Rannacher [42], Adaptive finite element methods for low Mach-number flows with chemical reactions.

Dedicated to Wolfgang Dahmen on the Occasion of his 60th Birthday

Nonlinear problems. IV

Adaptive Finite Element Methods for Contact Problems Embedded in a Fictitious Domain

Adaptive Finite Element Methods for the Euler Equations

Adaptive Finite Element Methods for Optimal Control Problems

During the last years, scientific computing has become an important research branch located between applied mathematics and applied sciences and engineering. Highly efficient numerical methods are based on adaptive methods, higher order discretizations, fast linear and non-linear iterative solvers, multi-level algorithms, etc. Such methods are integrated in the adaptive finite element software ALBERTA. It is a toolbox for the fast and flexible implementation of efficient software for real life applications, based on modern algorithms. ALBERTA also serves as an environment for improving existent, or developing new numerical methods in an interplay with mathematical analysis and it allows the direct integration of such new or improved methods in existing simulation software.

Finite Element Methods are used for numerous engineering applications where numerical solutions of partial differential equations are needed. As computers can now deal with the millions of parameters used in these methods, automatic error estimation and automatic adaptation of the utilised method (according to this error estimation), has become a hot research topic. This text offers comprehensive coverage of this new field of automatic adaptation and error estimation, bringing together the work of eight outstanding researchers in this field who have completed a six year national research project within the German Science Foundation. The result is a state-of-the-art work in true reference style. Each chapter is self-contained and covers theoretical, algorithmic and software presentations as well as solved problems. A main feature consists of several carefully elaborated benchmarks of 2D- and 3D- applications. * First book to go beyond the Finite Element Method in itself * Covers material from a new research area * Presents benchmarks of 2D- and 3D- applications * Fits with the new trend for genetic strategies in engineering

The Finite Element Toolbox ALBERTA

Computing with hp-ADAPTIVE FINITE ELEMENTS

Adaptive Finite Element Methods for Flow Problems in Regions with Moving Boundaries

Adaptive Finite Element Methods for Parabolic Partial Differential Equations

Adaptive Finite Element Methods for Shells

""Based on the proceedings of the first conference on superconvergence held recently at the University of Jyvaskyla, Finland. Presents reviewed papers focusing on superconvergence phenomena in the finite element method. Surveys for the first time all known superconvergence techniques, including their proofs.

The Sixth Edition of this influential best-selling book delivers the most up-to-date and comprehensive text and reference yet on the basis of the finite element method (FEM) for all engineers and mathematicians. Since the appearance of the first edition 38 years ago, The Finite Element Method provides arguably the most authoritative introductory text to the method, covering the latest developments and approaches in this dynamic subject, and is amply supplemented by exercises, worked solutions and computer algorithms. The classic FEM text, written by the subject's leading authors Enhancements include more worked examples and exercises With a new chapter on automatic mesh generation and added materials on shape function development and the use of higher order elements in solving elasticity and field problems Active research has shaped The Finite Element Method into the pre-eminent tool for the modelling of physical systems. It maintains the comprehensive style of earlier editions, while presenting the systematic development for the solution of problems modelled by linear differential equations. Together with the second and third self-contained volumes (0750663219 and 0750663227), The Finite Element Method Set (0750664312) provides a formidable resource covering the theory and the application of FEM, including the basis of the method, its application to advanced solid and structural mechanics and to computational fluid dynamics. The classic introduction to the finite element method, by two of the subject's leading authors Any professional or student of engineering involved in understanding the computational modelling of physical systems will inevitably use the techniques in this key text

The Finite Element Method: Its Basis and Fundamentals

Adaptive Finite Element Methods for the Obstacle Problem

Adaptive H-p Finite Element Methods for the Numerical Simulation of Crack Propagation

Adaptive Finite Element Methods for LES

P-adaptive and Automatic Hp-adaptive Finite Element Methods for Elliptic Partial Differential Equations

In this dissertation, we formulate and implement p-adaptive and hp-adaptive finite element methods to solve elliptic partial differential equations. The main idea of the work is to use elements of high degrees solely (p-adaptive) or in combination with elements of small size (hp-adaptive) to better capture the behavior of the solution. In implementing the idea, we deal with different aspects of building an adaptive finite element method, such as defining basis functions, developing algorithms for adaptive meshing procedure and formulating a posteriori

error estimates and error indicators. The basis functions used in this work are regular nodal basis functions and special basis functions defined for elements with one or more edges of higher degree transition elements). It is proved that with our construction of these basis functions, the finite element space is well-defined and C0. Several algorithms are developed for different scenarios of the adaptive meshing procedure, namely, p-refinement, p-unrefinement and hp-refinement. They all follow the 1-irregular rule and 2-neighbor rule motivated by [Bank and Sherman, 1983 - MR751598]. These rules help to limit the number of special cases and maintain the sparsity of the stiffness matrix, and thus to simplify the implementation and reduce the cost of calculation. The work of formulating a posteriori error estimates and error indicators is the core of this dissertation. Our error estimates and error indicators are based on the derivative recovery technique proposed by [Bank and Xu, 2003 - MR2034616, MR2034617] and Bank et al., 2007 - MR2346369]. Using the information in formulating the error indicators, we define a hp-refinement indicator which can be utilized to decide whether a given element should be refined in h or in p. Numerical results show that the combination of the two indicators helps automatic hp-refinement to create optimal meshes that demonstrate exponential rate of convergence. In this dissertation, we also consider hp-adaptive and domain decomposition when they are combined using the parallel adaptive meshing paradigm developed by [Bank and Holst, 2000 - MR1797889]. Numerical experiments demonstrate that the paradigm scales up to at least 256 processors (maximum size of our experiments) and with nearly 200 millions degrees of freedom.
The goal of this work is to develop robust schemes for the simulation of crack propagation using adaptive finite element analysis. Here we address the main problem of estimation of the error in the engineering quantity of interest (e.g., stress intensity factors for the crack, the stresses in the hot spots). We showed that the error in the quantity of interest can be estimated by employing the standard error estimators, for the energy norm of the error, which are existing in several commercial finite element programs. We have developed a computer based approach for checking the quality of the error estimators used in practice and we have also developed error estimators which are robust for meshes used in engineering practice. This work has led to an innovative approach for adaptive mesh refinement to obtain the quantity of interest with prescribed tolerance.

Finite Element Methods

Superconvergence, Post-Processing, and A Posterior Estimates

Adaptive Finite Element Methods for the Identification of Distributed Parameters in Partial Differential Equations

Adaptive Finite Element Methods for Optimization in Partial Differential Equations

Design of Adaptive Finite Element Software

In my dissertation, I developed mixed hp-finite element methods for linear elasticity with weakly imposed symmetry, which is based on Arnold-Falk-Winther's stable mixed finite elements. I have proved the h-stability of my method for meshes with arbitrary variable orders. In order to show the h-stability, I need an upper limit of the highest order of meshes, which can be an arbitrary nonnegative integer.

The key issues are a posteriori error estimation and it automatic mesh adaptation. Besides the traditional approach of energy-norm error control, a new duality-based technique, the Dual Weighted Residual method for goal-oriented error estimation, is discussed in detail. This method aims at economical computation of arbitrary quantities of physical interest by properly adapting the computational mesh. This is typically required in the design cycles of technical applications. For example, the drag coefficient of a body immersed in a viscous flow is computed, then it is minimized by varying certain control parameters, and finally the stability of the resulting flow is investigated by solving an eigenvalue problem. `Goal-oriented' adaptivity is designed to achieve these tasks with minimal cost. At the end of each chapter some exercises are posed in order to assist the interested reader in better understanding the concepts presented. Solutions and accompanying remarks are given in the Appendix.

Adaptive Finite Element Methods for Differential Equations

Adaptive finite element methods for variational inequalities

Adaptive Finite Element Methods for Nonlinear Hyperbolic Problems of Second Order

Multiscale and Adaptivity: Modeling, Numerics and Applications

Computation of the Mean Drag Coefficient in a Turbulent Flow Around a Surface Mounted Cube Using Adaptive Mesh Refinement

This dissertation, "Adaptive Finite Element Analysis for 2D Elastostatic Problems" by [illegible], Chi-king, Lee, was obtained from The University of Hong Kong (Pokfulam, Hong Kong) and is being sold pursuant to Creative Commons: Attribution 3.0 Hong Kong License. The content of this dissertation has not been altered in any way. We have altered the formatting in order to facilitate the ease of printing and reading of the dissertation. All rights not granted by the above license are retained by the author. Abstract: Abstract of Thesis Entitled: "Adaptive Finite Element Analysis For 2D Elastostatic Problems" submitted by LEE CHI KING for the degree of Master of Philosophy at The University of Hong Kong in October of 1992 iii Abstract The objective of an adaptive

finite element procedure is to model a physical problem by the finite element method and to achieve a solution of specified accuracy in a most economic manner. A robust and fully automatic adaptive finite element procedure is a very powerful and useful tool to practical engineers. Through the adaptive process the user can, simultaneously, obtain an accurate solution of the physical problem and also a reliable estimate of the error of the finite element solution. This allows engineers and designers to have a better understanding of the behavior of the physical structure, and to optimize their designs and obtain more economical and safer designs. In this thesis, attentions will be paid to the adaptive refinement process for solving 2D elastostatic problems, in which three essential steps can be identified. These steps are development of, (i) an economical and reliable error estimation procedure which predicts the numerical error of the finite element method, (ii) an effective methodology for predicting and defining the necessary refinements needed to achieve the prescribed accuracy in an optimal manner, and (iii) a practical algorithm for the construction of the refined finite element model with specified mesh density and grading by means of an automatic mesh generation scheme. A new adaptive refinement procedure which integrates the latest advanced techniques developed in the above three aspects will also be proposed. This new refinement procedure will employ the Zienkiewicz and Zhu error estimator for predicting the error of the finite element solution. A new algorithm, based on the strain energy density concentration concept for automatically locating any singular points in the problem domain, is developed for defining the optimal mesh pattern for the refinement process. Three new automatic mesh generators which are capable of generating either pure triangular, mixed or pure quadrilateral meshes are developed iv for generating high quality, well-graded finite element meshes with node spacing closely compatible with the required pattern for optimal convergence of the solution. The performance and robustness of the proposed adaptive refinement scheme are tested by applying it to solve practical engineering problems with different geometrical complexity and difficulty. Finally, potential and possible areas for further extension of the current research work in different aspects of finite element applications will be suggested. DOI: 10.5353/th_b3101931 Subjects: Finite

element method Elasticity - Mathematical models

An adaptive finite element procedure is developed for the transient analysis of nonlinear shells. The scheme is an h-method which employs fission and fusion. Criteria based on incremental work and deviation of the bilinear finite element approximation to the shell from a Kirchhoff-Love surface are used as criteria for adaptivity. The example problems show that the adaptive schemes are capable of achieving substantial improvements in accuracy for a given computational effort. They include both material and geometric nonlinearities and local and global buckling. In order to formulate an r-adaptive method, the conservation laws, the constitutive equations, and the equation of state for path-dependent materials are formulated for an arbitrary Lagrangian-Eulerian description. Both geometrical and material nonlinearities are included in this setting. Keywords: Finite elements, Adaptive meshes, Shells.

C.I.M.E. Summer School, Cetraro, Italy 2009

Mixed Hp-adaptive Finite Element Methods for Elasticity and Coupled Problems

Error Controlled Hp-Adaptive Finite Element Methods for the Time-Dependent Maxwell Equations

Adaptive Finite Element Methods for Reactive Compressible Flow

The book of invited articles offers a collection of high-quality papers in selected and highly topical areas of Applied and Numerical Mathematics and Approximation Theory which have some connection to Wolfgang Dahmen's scientific work. On the occasion of his 60th birthday, leading experts have contributed survey and research papers in the areas of Nonlinear Approximation Theory, Numerical Analysis of Partial Differential and Integral Equations, Computer-Aided Geometric Design, and Learning Theory. The main focus and common theme of all the articles in this volume is the mathematics building the foundation for most efficient numerical algorithms for simulating complex phenomena.

Multiscale, Nonlinear and Adaptive Approximation

Convergence of Goal-oriented Adaptive Finite Element Methods

Adaptive Finite Element Methods for the Unsteady Maxwell's Equations